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13. ABSTRACT (Maximum 200 words)

The program covered a great variety of problems. Consider the allocation of the base station transmitter in time varying mobile communications with many (data) users. The channel rates for the various users are estimated at the start of the small scheduling intervals. Since the rates vary randomly, there is a conflict between full use (by selecting the user with the highest current rate) and fairness. The Proportional Fair Scheduler is designed to deal with such conflicts. Convergence and optimality properties were shown, and the algorithm extended to cover a much greater variety of circumstances.
Consider the forward link of a mobile communications system with randomly time varying channels connecting the to the moblies. Data arrives at the base in some random way and is queued. Stochastic stability methods are used to allocation transmitter power and time to the various queues in a queue- and channel-state dependent way to assure stability and good operation. Most of the schemes of current interest can be handled such as various forms of CDMA and TDMA, including multiantenna versions.
General occupancy models were applied to the problem of blocking in optical communications switches. Since the probabilities are small, large deviations methods are appropriate. A complete large deviation analysis of this problem is carried out, and the results yield accurate estimates.

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# Stochastic Control problems in Mobile Communications
# Final Report, Army Research office, Dec. 2002
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- **Convergence of Proportional-Fair Sharing Algorithms Under General Conditions [1].** We were concerned with the allocation of the base station transmitter time in time varying mobile communications with many users who are transmitting data. Time is divided into small scheduling intervals, and the channel rates for the various users are available at the start of the intervals. Since the rates vary randomly, there is a conflict between full use (by selecting the user with the highest current rate) and fairness. The Proportional Fair Scheduler (PFS) of the Qualcomm High Data Rate (HDR) system and related algorithms are designed to deal with such conflicts. The aim was to put such algorithms on a sure mathematical footing and analyze their behavior. Such algorithms are of the stochastic approximation type and results of stochastic approximation are used to analyze the long term properties. It was shown that the limiting behavior of the sample paths of the throughputs is well approximated by the solution of an intuitively reasonable ordinary differential equation, which is akin to a mean flow. It was shown that the ODE has a unique equilibrium and that it is characterized as optimizing a concave utility function, which shows that PFS is not ad-hoc, but actually corresponds to a reasonable maximization problem. These results may be used to analyze the performance of PFS. The results depend on the fact that the mean ODE has a special form that arises in problems with certain types of competitive behavior. There is a large set of such algorithms, each one corresponding to a concave utility function. This set allows a choice of tradeoffs between the current rate and throughout. Extensions to multiple antenna and frequency systems are given. Finally, the infinite backlog assumption is dropped and the data is allowed to arrive at random. This complicates the analysis, but the same results hold.

- **Wireless systems with time varying channels [2].** Consider the forward link of a system with $K$ remote units and a single base transmitter with time varying connecting channels. Data to be transmitted to the remote units arrives according to some random process and is queued according to its destination. Power is to be allocated to the $K$ channels in a queue and channel state dependent way to minimize some cost criterion. The channel fading rate is fast and the bandwidth and data arrival rates are high. Owing to the high speed the fading, arrival, and service rates,

an asymptotic or averaging of the heavy traffic type method is promising. By heavy traffic, we mean that on the average there is little server idle time and spare power over the "average" requirements. Heavy traffic analysis has been very helpful in simplifying control problems in queueing and communications networks. It eliminates inessential detail and focuses on the fundamental issues of scaling and parametric dependencies. To illustrate the scope of the method, several models are considered. The basic model assumes that the channel state is known, and that given the channel state there is a well defined rate of transmission per unit power. Then convergence of the controlled scaled queue lengths is shown. The scaling is different from the usual in heavy traffic work. The appropriate orders of reserve power and buffer size are given as well as suggested policies. The approximating process is a controlled reflected diffusion which is simpler than the original problem and facilitates understanding parametric dependencies, solutions and stability. The averaging is robust and can be done under a great variety of conditions. To illustrate the scope of the method, more complicated systems are also treated; e.g, power scheduling is done at prescribed intervals only, knowledge of the channel might be subject to random errors, retransmissions might be needed due to excessive errors at the receiver, or the system might be of the TDMA type. The methods and approximating control problems are similar.

- **Stability and Control of Mobile Communications Systems With Time Varying Channels [3].** Consider the forward link of a mobile communications system with a single transmitter and rather arbitrary randomly time varying channels connecting the base to the mobiles. Data arrives at the base in some random way and is queued according to the destination until transmitted. The main issues are the allocation of transmitter power and time to the various queues in a queue- and channel-state dependent way to assure stability and good operation. The control decisions are made at the beginning of the (small) scheduling intervals. Stability methods are used to allocate time and power. Many schemes of current interest can be handled: For example, CDMA with control over the bit interval and power per bit, TDMA with control over the time allocated, power per bit, and bit interval, as well as arbitrary combinations. The details of the scheme are not directly involved; all essential factors are incorporated into a "rate" and "error" function. There might be random errors in transmission which require retransmission. The channel-state process might be known or only partially known. The system and channel process are scaled by speed. A fluid approximation is derived for a canonical model, via a weak convergence analysis. Under a stability assumption on the fluid model and some other natural conditions, it is shown that the physical system can be controlled to be stable, uniformly in the speed. Owing to the non-Markov nature of the problem, we use the perturbed Liapunov function method, which is very useful for the analysis of non-Markovian systems. Finally, the perturbed Liapunov method is used to

actually choose the power and time allocations. The allocation will depend on the Liapunov function. But each such function corresponds loosely to an optimization problem for some performance criterion. Since there is a choice of Liapunov functions, various performance criteria can be taken into account in the allocations. The power of the method is due to the rather general conditions under which it works.

- **Large Deviation Approximations for Occupancy Problems.** The paper [4] was completed. In the occupancy problem one considers the distribution of $r$ balls in $n$ cells, with each ball assigned independently to a given cell with probability $1/n$. In the paper just mentioned a large deviation approximation as $r$ and $n$ tend to infinity was proved. Occupancy problems have many applications in computer science, mathematical biology and elsewhere, but the original motivation for this work is the design of an optical communication switch, where blocking probabilities are of central importance. Here the urns correspond to channels and the balls to packets being routed through the switch. In order to analyze the problem a dynamical model is first considered, where the balls are placed in the cells sequentially and "time" corresponds to the number of balls that have already been thrown. A complete large deviation analysis of this "process level" problem is carried out, and the rate function for the original problem is then obtained via the contraction principle. The variational problem that characterizes this rate function is analyzed, and (in sharp contrast to most analyses of large deviations for Markov processes), a complete and explicit solution is obtained. The minimizing trajectories and minimal cost are identified up to two constants, and the constants are characterized as the unique solution to an elementary fixed point problem. These results are then used to solve a number of interesting problems, including the overflow problem and the so-called partial coupon collector's problem.

  Extensions of this work will consider urn problems with balls of differing colors. One can define a mapping that turns the occupancy process for colored balls into a process for random motion on a lattice, which we refer to as a migration process. With some further extension of the underlying extremals, the corresponding solutions to the variational problems for the migration process can be identified. Migration processes are relevant to wireless networks, in particular to ad hoc networks where the statistical properties of the mobile node positions with respect to one another are related to the communication capabilities of the network.

- **Regulation and Analysis of Stochastic Networks.** Research in this area proceeded along several directions. Key issues in our investigations included: (i) the development of approximate models that allow for explicit (or nearly explicit) construction of the optimal routing/service policies, and (ii) robustness and the ability to deal with model perturbations. The problems considered encompass a number of different formulations,

each of which emphasizes different qualitative properties of the resulting controlled network. These include risk-sensitive control and the control of rare events, optimal control of "fluid" models, and optimally robust control of such models. "Fluid" models are approximate models that are obtained under a law of large numbers scaling. In all cases the system model is constrained, and in most problems this is a dominant feature. To handle the constraints, we model the state dynamics of the approximate (or limit) models in terms of an appropriate Skorokhod Problem and corresponding constrained ordinary or stochastic differential equations. The different problem setups (e.g., control of rare events versus control of fluid models) lead to problems of the same basic form, which is a Skorokhod Problems with (relatively) simple dynamics and simple cost structures.

Paper [5] considers the problem of characterizing the manner in which rare events occur in networks under heavy traffic. It shows how time reversal arguments can be applied to rewrite the control problem defined by a straightforward large deviations analysis into the form of [5], and then shows how explicit finite-dimensional solutions can be obtained for some interesting classes of problems (in arbitrary dimension). A three dimensional example is worked out in detail for illustrative purposes.

The paper [6] considers the robust optimal control of a law of large numbers approximation of a stochastic network. The robust control problem is formulated as a dynamic differential game, with one player choosing the policies that determine service and routing assignments, and the other choosing quantities such as the arrival and service rates, subject to constraints. The cost to be minimized by the first player and maximized by the second is the time till the origin is reached. The robust formulation allows one to differentiate between the many policies (at the fluid level) that are optimal for an ordinary cost. An explicit formula is given for the value function, and some of its basic properties are studied. The problem of policy synthesis for particular classes of problems has also been studied, and for these cases we have explicitly constructed a robustly optimal control.

- **Importance sampling and simulation.** The design of efficient simulation schemes is an important consideration in the design and analysis of stochastic systems and stochastic networks. The paper [7] considers fundamental issues related to the technique known as "importance sampling." A standard heuristic for importance sampling is that the changes of measure used to prove large deviation lower bounds give good performance when used for importance sampling. Recent work, however, has suggested that the heuristic is incorrect in many situations. The perspective put forth in [7] is that large deviation theory suggests many changes of measure, and that not all are suitable for importance sampling. In the setting of Cramér's Theorem, the traditional interpretation of the heuristic

suggests a fixed change of distribution on the underlying independent and identically distributed summands. In contrast, [7] considers importance sampling schemes where the exponential change of measure is adaptive, in the sense that it depends on the historical empirical mean. The existence of asymptotically optimal schemes within this class is demonstrated. The result indicates that an adaptive change of measure, rather than a static change of measure, is what the large deviations analysis truly suggests. The proofs utilize a control-theoretic approach to large deviations, which naturally leads to the construction of asymptotically optimal adaptive schemes in terms of a limit Bellman equation. Numerical examples contrasting the adaptive and standard schemes are presented, as well as an interpretation of their different performances in terms of differential games.

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